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Designing a quality gain-loss function for smaller-the-better characteristic under not neglecting the linear term loss[1]

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Abstract. Because the smaller-the-better quality characteristic cannot achieve zero in practice, it is unreasonable to express the quality gain-loss function only by the compensation and the loss of quadratic term, and it is unreasonable to directly delete the loss of the linear term, therefore a new quality gain-loss function for smaller-the-better characteristic is designed. When the linear term loss is not ignored and the compensation amount is kept constant, the expression form of the smaller-the-better gain-loss function is studied, the determination method for the loss coefficient of the linear term and quadratic term in quadratic quality gain-loss function is researched, and the linear term and the quadratic term loss in quadratic gain-loss function are compared and analyzed.

Key words. The smaller-the-better characteristic, gain-loss function, loss coefficient, compensation amount.

1. Introduction

Ever since the quality loss function was proposed by Taguchi, many scholars have done a lot of research about product quality loss model. Aimed at the limitations of the quality loss function, the inverted-normal distribution functions were used to solve an unbounded quality loss function [1]. For the weight-loss problem of asymmetry, an asymmetric quality loss function model was proposed using the theory of

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piecewise functions to extend the quality loss function, and established the quality loss function model [2–5]. Fuzzy logic was used to present the concept of fuzzy quality loss, and the fuzzy quality loss function model was established [6]. While most studies have focused on single characteristics of the quality loss function, a multiple quality characteristics model of total mass loss and the method of tolerance design were presented [7]. The method of scaling and multivariate Taylor series expansion of the function to propose a more general form of multiple-parameter quality loss model was applied [8, 9]. Because the smaller-the-better quality characteristic cannot achieve zero in practice, a quadratic quality loss function for the smaller-the-better characteristic under not neglecting the linear term loss was proposed [10]. Because the quality loss function could not describe the quality compensation effect in production practice, due to its giving the constant term in the Taylor series expansion a physical meaning—the quality of compensation—the concept of quality gain-loss function was proposed and the quality gain-loss model of transmitting and a method of tolerance optimization for quality characteristics were studied[11], [12].

In the literature of studying on quality gain-loss function, the quality loss was almost represented only by a quadratic term, it not only neglects the linear terms but also the higher-order terms. It is possible to express the quality gain-loss function only with a quadratic term for target characteristic, but this is inappropriate for the smaller-the-better characteristic. For the quality characteristic target value can be realized and the minimum can be reached. When the compensation amount is kept constant, the quality gain-loss function for the smaller-the-better characteristic is the Taylor expansion of the quality characteristic value at zero quality characteristic point. According to the core idea of the quality gain-loss function, the quality loss is 0 when the quality characteristic value reaches zero quality characteristic point, and the first derivative is 0 for the quality loss value reaching the minimum as the quality characteristic value goes to zero quality characteristic point. Although the quality characteristics value does not necessarily reach zero quality characteristic point, the quality characteristic value is relatively small and the higher-order terms above the second order are relatively small, so the higher order terms can be omitted. Thus, there is only a constant term and a quadratic term in the Taylor expansion, i.e., the quality gain-loss function is represented only by the constant term and the quadratic term.

The smaller the smaller-the-better quality characteristic, the better, but the product quality characteristic value is always finite in practice, neither the zero quality characteristic point nor the minimum can be reached. Therefore, the first derivative of Taylor expansion is not equal to 0. It is unreasonable to directly delete the linear term loss of the quality gain-loss function. In this paper, the quality gain-loss function is appropriately expressed in the quadratic term under not neglecting the linear term loss function and keeping the compensation amount constant, and the determination method for the loss coefficient of the linear term and the quadratic term in quadratic quality gain-loss function is proposed simultaneously.

2. The quality gain-loss function for the smaller-the-better characteristic

Assume that the quality characteristic value of a product is y and the quality gain-loss function corresponding to quality characteristic value y is $G(y)$. The smaller-the-better characteristic quality gain-loss function comes from the Taylor expansion of $G(y)$ at point $y = 0$ [1]

$$G(y) = G(0) + \frac{G'(0)}{1!}y + \frac{G''(0)}{2!}y^2 + o(y^2)\,. \tag{1}$$

Assuming that the quality compensation is constant, for the smaller-the-better quality characteristics, the quality gain-loss reaches the minimum when the quality characteristic value reaches zero. In other word, $G(0) = \sigma$, $\sigma \in R$. Because the quality gain-loss reaches the minimum at 0, $G'(0) = 0$. Omitting the higher-order terms above the second order, we obtain

$$G(y) = G(0) + k\,y^2\,. \tag{2}$$

The smaller the smaller-the-better characteristic, the better, but the smaller-the-better quality characteristic value cannot really reach zero and the maximum point (zero point) cannot be reached in practice. Therefore we cannot directly let $G'(0) = 0$ in (1), as $G(\infty) \neq 0$. In other words, the loss of the linear term is not negligible. Since the loss of the linear term is not ignored directly, the quality gain-loss function should be expressed in quadratic form.

In the model of the quadratic quality gain-loss function, an intersection exists between the linear term function curve and the quadratic term function curve, so that the amount of the linear term loss and quadratic term loss can change with the changes of the quality characteristic value. Then, make the linear term quality loss function $G_1(y) = k_1/y$ and the quadratic term quality loss $G_2(y) = k_2/y$. The relationship between the linear and quadratic term quality loss functions for the smaller-the-better characteristic is shown in Fig. 1.

It can be seen in Fig. 1 that when $y < y_0$, the loss of the linear term is higher than that of quadratic term; when $y > y_0$, the loss of the linear term is less than that of quadratic term. And when $y = y_0$, the loss of the linear term is equal to that of the quadratic term. From Fig. 1 we can see that only when $y < y_0$ reaches a certain value and the loss of the linear term is far less than that of the quadratic term loss, the loss of the linear term can be ignored.

3. The design of quadratic form quality gain-loss function for the smaller-the-better characteristic

In the Taylor expansion of the quality gain-loss function, although the linear term quality loss cannot be ignored directly, for the smaller-the-better quality characteristic the quality characteristic value is small and the higher-order terms above the second order are very small, so the higher-order terms can be omitted. Because

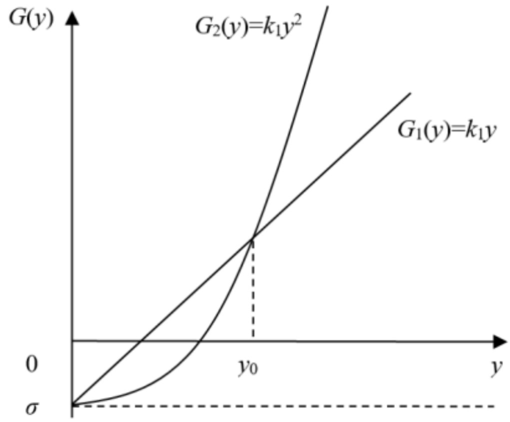

Fig. 1. The relationship between the linear and quadratic term quality loss functions for the smaller-the-better characteristic

the quality compensation quantity is constant, the quality gain-loss function for the smaller-the-better characteristic is:

$$G(y) = \sigma + k_1\, y + k_2\, y^2\,. \tag{3}$$

When the compensation quantity is assumed to be constant in the theory of quality gain-loss function, the greater the deviation of the quality characteristic value from the target value, the greater the quality gain-loss, so $k_1 \geq 0$, $k_2 > 0$ and $\sigma \in R$. The product is defective when the quality characteristic value exceeds the specification limit (tolerance Δ). In this case, the required cost of reworking or repairing the product is A. The product losses are now determined by the function when the quality characteristic value exceeds the functional limit (i.e., the deviation is greater than Δ_0); in this case the quality gain-losses caused by the product being scrapped is A_0. Therefore, by formula (3) we get

$$A = \sigma + k_1\, \Delta + k_2\, \Delta^2$$

and

$$A_0 = \sigma + k_1\, \Delta_0 + k_2\, \Delta_0^2\,. \tag{4}$$

From (4) we have

$$k_1 = \frac{(A-\sigma)\Delta_0^2 - (A_0-\sigma)\Delta^2}{\Delta\Delta_0(\Delta_0-\Delta)}$$

and

$$k_2 = \frac{(A_0-\sigma)\Delta - (A-\sigma)\Delta_0}{\Delta\Delta_0(\Delta_0-\Delta)}\,. \tag{5}$$

Putting both equations (5) into (3), we obtain the quality gain-loss function in the form

$$G(y) = \sigma + \frac{(A-\sigma)\Delta_0^2 - (A_0-\sigma)\Delta^2}{\Delta\Delta_0(\Delta_0-\Delta)}y + \frac{(A_0-\sigma)\Delta - (A-\sigma)\Delta_0}{\Delta\Delta_0(\Delta_0-\Delta)}y^2 . \qquad (6)$$

To analyze the effect of the quality gain-loss function for the smaller-the-better characteristic, the quadratic form quality gain-loss function for the smaller-the-better characteristic can be divided into linear term loss, quadratic term loss, and constant term compensation as

$$G_1(y) = \frac{(A-\sigma)\Delta_0^2 - (A_0-\sigma)\Delta^2}{\Delta\Delta_0(\Delta_0-\Delta)}y , \qquad (7)$$

$$G_2(y) = \frac{(A_0-\sigma)\Delta - (A-\sigma)\Delta_0}{\Delta\Delta_0(\Delta_0-\Delta)}y^2 , \qquad (8)$$

$$G_3(y) = \sigma . \qquad (9)$$

The ratio of the linear and the quadratic term equals π, so that

$$\pi = \frac{G_1(y)}{G_2(y)} = \frac{(A-\sigma)\Delta_0^2 - (A_0-\sigma)\Delta^2}{(A_0-\sigma)\Delta - (A-\sigma)\Delta_0}\,\frac{1}{y} =$$

$$= \frac{\left(\frac{\Delta_0}{\Delta}\right)^2 - \left(\frac{A_0-\sigma}{A-\sigma}\right)^2}{\frac{A_0-\sigma}{A-\sigma} - \frac{\Delta_0}{\Delta}}\,\frac{\Delta}{y} . \qquad (10)$$

According to the value of π in equation (10), the following cases were discussed:

When $\pi = 1$, $G_1(y) = G_2(y)$. Let $y = y_0$ be the point of intersection of the linear and quadratic functions in Fig. 1, that is

$$y_0 = \frac{\left(\frac{\Delta_0}{\Delta}\right)^2 - \left(\frac{A_0-\sigma}{A-\sigma}\right)^2}{\frac{A_0-\sigma}{A-\sigma} - \frac{\Delta_0}{\Delta}}\,\Delta .$$

.

At this point, whether the loss of the linear term relative to the quadratic term is negligible, depends on the value of y_0. For the smaller-the-better quality characteristic, if y_0 is very small, that is $1/y_0 \to \infty$, then the linear term loss is smaller than that of the quadratic term, and the linear term loss is negligible. Otherwise, it is not negligible.

When

$$\frac{A_0-\sigma}{A-\sigma} \to \left(\frac{\Delta}{\Delta_0}\right)^2 ,$$

then $\pi \to 0$, so that the linear term loss is negligible compared with the quadratic term loss, and the intersection of the linear term function curve and the quadratic function curve goes to zero $y_0 \to 0$.

When

$$\frac{A_0 - \sigma}{A - \sigma} \to \frac{\Delta}{\Delta_0}$$

and $\pi \to \infty$, then y_0 is high and the linear term loss is greater than the quadratic term loss. Then, the intersection of the linear term function and quadratic term function y_0 is very high, which is the most common situation in practice. Although the consumers expect the smaller-the-better quality characteristic value, and the smaller the better, the actual value of the characteristic is not small, and the linear term loss cannot be neglected at this time. The linear term can be ignored, as shown in Fig. 2, and the linear term cannot be ignored, as shown in Fig. 3.

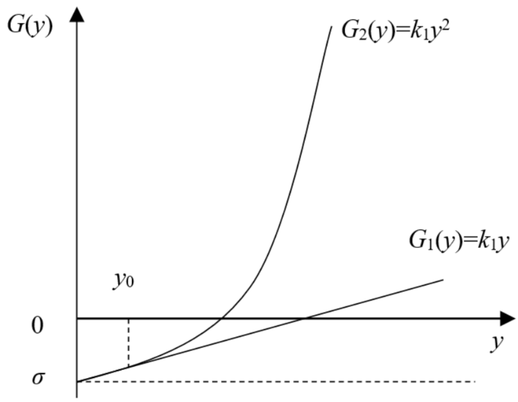

Fig. 2. The linear term can be ignored

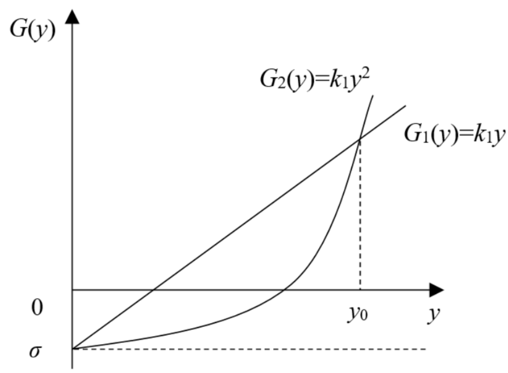

Fig. 3. The linear term cannot be ignored

When

$$\frac{\Delta}{\Delta_0} < \frac{A_0 - \sigma}{A - \sigma} < \left(\frac{\Delta}{\Delta_0}\right)^2,$$

then the ratio $G_1(y)/G_2(y)$ can determine whether the linear term can be neglected. Substitute $\bar{y}$ for y in (10), so that the equation now reads

$$\frac{G_1(y)}{G_2(y)} = \frac{\left(\frac{\Delta_0}{\Delta}\right)^2 - \left(\frac{A_0-\sigma}{A-\sigma}\right)^2}{\frac{A_0-\sigma}{A-\sigma} - \frac{\Delta_0}{\Delta}} \frac{\Delta}{\bar{y}} = \pi\,. \tag{11}$$

For the smaller-the-better quality characteristic, remove the human factors, which means $\bar{y} < \Delta$ under normal circumstances. When the actual value of π is very small, generally because the linear term loss is much less than the quadratic term loss, then the linear term loss can be neglected. Otherwise, it cannot be neglected. In the actual application process, a specific criterion can also be determined: the linear term loss can be neglected if it is less than 10 % of the quadratic term loss

$$\frac{A-\sigma}{A_0-\sigma} = \left(\frac{\Delta}{\Delta_0}\right)^2 .$$

Since $k_1 \geq 0$ and $k_2 > 0$ in the equation (3), then

$$(A-\sigma)\Delta^2 - (A_0-\sigma)\,\Delta_0^2 \geq 0\,,$$

and

$$(A_0-\sigma)\Delta_0 - (A-\sigma)\,\Delta > 0\,.$$

Hence

$$\frac{A_0-\sigma}{A-\sigma} \leq \left(\frac{\Delta}{\Delta_0}\right)^2 \text{ and } \frac{\Delta}{\Delta_0} < \frac{A_0-\sigma}{A-\sigma}\,.$$

In the actual application process, if the parameter is reasonable, then

$$\frac{A_0-\sigma}{A-\sigma} > \left(\frac{\Delta}{\Delta_0}\right)^2$$

or

$$\frac{\Delta}{\Delta_0} \geq \frac{A_0-\sigma}{A-\sigma}$$

cannot occur.

When $k_1 \geq 0$ in equation (3), it may happen that $k_1 = 0$. In such a case

$$\frac{A-\sigma}{A_0-\sigma} = \left(\frac{\Delta}{\Delta_0}\right)^2$$

. Now, the quality gain-loss function for the smaller-the-better characteristic is

$$G(y) = \sigma + k\,y^2 \tag{12}$$

In the classic quality gain-loss function where the compensation is constant, as shown in equation (12), the tolerance of quality characteristic can be determined by the functional limit, and the quality gain-loss caused by discarded product is

$$\Delta = \sqrt{\frac{A-\sigma}{A_0-\sigma}}\,\Delta_0\,. \tag{13}$$

4. The analysis of practical problems

To show that a quality gain-loss function for the smaller-the-better characteristic under not neglecting the linear term loss and keeping the compensation amount constant is superior to the quadratic term quality gain-loss function, it is good to analyze a concrete case.

A dam concrete construction project mainly includes concrete production, concrete transportation, concrete pouring and concrete maintenance. Among them, the key quality indicators of concrete production have the outlet temperature of the concrete mixture (unit: days), it is the smaller-the-better characteristic. Assume that the design target value of the concrete mixture outlet temperature is 7 °C, and when deviation from the design target temperature value $y \geq 2$°C, the product is not qualified. That is, the design target temperature value tolerance $\Delta = 2$°C and the loss caused by it is 75 yuan/m^3. When the deviation from the design target temperature value $y \geq 5$°C, the products lose their function. That is, the functional limit of deviation from the design target temperature value $\Delta = 5$°C, and the loss caused by it is 200 yuan/m^3. Assume that the quality compensation of the next process for this process or the compensation produced by parallel processes through mutual cooperation $\sigma = -10\,\text{yuan/m}^3$. To evaluate the quality of concrete production, 10 concrete mixture samples were randomly selected during a particular period to obtain the deviation from the design target temperature in the quadratic term gain-loss function and the function under not neglecting the linear term gain-loss.

4.1. Evaluation using the quadratic term gain-loss function

When $\Delta = 2$°C, $A = 75\,\text{yuan/m}^3$, $\sigma = -10\,\text{yuan/m}^3$, the quadratic term gain-loss function is

$$G(y) = -10 + 21.5y^2\,. \tag{14}$$

The number of the sample is used in equation (14), and then the average of it is determined to obtain the average of quality gain-loss: $G_{\mathrm{a}} = 11.9\,\text{yuan/m}^3$.

4.2. Evaluation using the function under not neglecting the linear term gain-loss

When $\Delta = 2$°C, $A = 75\,\text{yuan/m}^3$, $\Delta_0 = 5d$, $A_0 = 200\,\text{yuan/m}^3$, $\sigma = -10\,\text{yuan/m}^3$, the function under not neglecting the linear term gain-loss is

$$G(y) = -10 + 37.5y + 2.5y^2\,. \tag{15}$$

The number of the sample is used in equation (15), then its average is determined to obtain the average of quality gain-loss: $G_b = 25.6\,\text{yuan}/\text{m}^3$, the average linear term loss, $L_1 = 33\,\text{yuan}/\text{m}^3$, and the average of quadratic term loss $L_2 = 2.6\,\text{yuan}/\text{m}^3$.

When the quality compensation is constant, the quality gain-loss vale when maintaining the linear term is 13.7 yuan $/\text{m}^3$ more than the value of the quadratic term quality gain-loss function, because the quadratic term quality gain-loss function neglects the linear term loss. Judged on the amount of the linear term loss and the quadratic term loss of the average quality gain-loss function equation under not neglecting the linear term loss for the smaller-the-better characteristic, the quadratic term loss is less than the linear term loss, so the linear term loss cannot be neglected. Of course, when not neglecting the linear term loss, the loss coefficient equation of the linear term and the quadratic term has changed, so the corresponding quadratic term loss is not equal to the quality gain-loss expressed only by the quadratic loss function.

5. Conclusion

When the quality gain-loss function is represented only by the compensation function and the quadratic term, it is not only neglects the linear term in the Taylor expansion but also the higher-order terms above the second order. However, it is not appropriate for the smaller-the-better characteristic to function like this. The quality gain-loss function for the smaller-the-better characteristic is studied from theoretical and practical perspectives in this paper, and the form of quadratic quality gain-loss function for the smaller-the-better characteristics under not neglecting the linear term loss and keeping the compensation amount constant and the corresponding loss coefficient equation of the linear term and quadratic term were put forward, and the amount of the linear term loss and the quadratic term loss for the smaller-the-better characteristics are compared. This study shows that the original quadratic term quality gain-loss function is a special case considering the linear term quality gain-loss function.

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